Inverse Problems and Related Fields

November 24-25, 2016

FRUMAM, Marseille, France

GDR CATIA   GDR DynQua

Aix*Marseille université   labex Archimède   CPT   INSTITUT DE MATHÉMATIQUES de MARSEILLE   FRU MAM

Presentation

The fourth edition of this two days workshop on **Inverse Problems**, **Mathematical Control**, and **Analysis of Partial Differential Equations**, will take place on 24 and 25 November 2016 in Marseille, France. This event is hosted by FRUMAM (Fédération de Recherche des Unités de Mathématiques de l'Agglomération Marseillaise), and is funded by the GOMS programme, Labex Archimède, Aix-Marseille Université, GDR CATIA, GDR DynQua, CPT (UMR 7332), and I2M (UMR 7373).

All talks will be given in the second floor's conference room of :

FRUMAM
3, place Victor Hugo
13331 Marseille.

Travel information can be found at www.frumam.cnrs-mrs.fr.

Speakers

Pedro CARO (Basque Center for Applied Mathematics, Bilbao, Spain)
Anabela DA SILVA (Aix-Marseille Université, France)
Thierry DAUDÉ (Université de Cergy-Pontoise, France)
Sylvain ERVEDOZA (Université Paul Sabatier, Toulouse, France)
Thomas GILETTI (Université de Lorraine, Nancy, France)
Manuel GONZÁLEZ BURGOS (Universidad de Sevilla, Spain)
Eva SINCICH (Università degli Studi di Trieste, Italy)
Marian SLODICKA (University of Ghent, Belgium)
Masahiro YAMAMOTO (Tokyo University, Japan)

Schedule

Day 1: Nov. 24

12:00 – 14:00	**Warm up.** Programme will begin at 14:00 sharp, but we plan to have lunch around 12:30, not too far from the conference room, so don't hesitate to join us.
14:00 – 15:00	**Thomas GILETTI** Propagating terraces: existence and properties.
15:00 – 16:00	**Pedro CARO** An inverse scattering problem in random media.
16:00 – 16:20	**Coffee Break**
16:20 – 17:20	**Anabela DA SILVA** (On mice and Men) Various Diffuse Optical Tomography techniques.
20:00	**Conference Dinner** at O_2' pointus "CNTL"

Day 2: Nov. 25

9:00 – 10:00	**Marian SLODICKA**
	Some inverse problems in evolutionary partial differential equations.
10:00 – 10:20	**Coffee Break**
10:20 – 11:20	**Eva SINCICH**
	Lipschitz stability for the electrostatic inverse boundary value problem with piecewise linear conductivities.
11:20 – 12:20	**Manuel GONZÁLEZ BURGOS**
	Controllability of linear parabolic systems : Hyperbolic phenomena.
12:20 – 14:00	**Lunch**
14:00 – 15:00	**Thierry DAUDÉ**
	Non-uniqueness results for the anisotropic Calderón problem with data measured on disjoint sets.
15:00 – 16:00	**Sylvain ERVEDOZA**
	On the recovery of coefficients in a wave equation.
16:00 – 16:20	**Coffee Break**
16:20 – 17:20	**Masahiro YAMAMOTO**
	Coefficient inverse problems for integro-partial differential equations by Carleman estimates.

Abstracts

Pedro CARO

An inverse scattering problem in random media.

Abstract. In inverse scattering theory the aim is to determine a scattering potential q from appropriate measurements. In many applications the scatterer is non-smooth and vastly complicated. For such scatterers, the inverse problem is not so much to recover the exact micro-structure of an object but merely to determine the parameters or functions describing the properties of the micro-structure. An example of such a parameter is the local strength of the scatterer, which shows how realizations oscillate around the mean. In mathematical terms, the potential q is assumed to be a Gaussian random function whose covariance operator is a classical pseudo-differential operator. We show that the backscattered field, obtained from a single realization of the random potential q, determines uniquely the principal symbol of the covariance operator. This is a joint work with Tapio Helin and Matti Lassas.

Anabella DA SILVA

(On mice and Men) Various Diffuse Optical Tomography techniques.

Abstract. Optical Tomography (DOT) techniques are in vivo imaging methods based on the exploitation of the visible to near infrared light radiation ($\sim 600 - 1000\ nm$). They allow to produce quantitative functional images of biological tissues with resolutions $\sim 1 - 5\ mm$ according to the depth, this one can be up to $\sim 10\ cm$. In this wavelength range, biological tissues can be modeled as weakly absorbing but highly scattering media (scattering mean free path $\sim 20 - 50\ \mu m$). Under these conditions, light propagation is dominated by multiple scattering and can be modeled under the form of the so-called Radiative Transport Equation (RTE), than can be approximated by a Diffusion Equation (DE) if the tissues are sufficiently large. The wide variety of detectors and sources available in this wavelength range, offers the possibility to play with spatial and time modulation to access to the quantitative estimation of the optical properties (absorption and scattering coefficients) of the tissues, and hence to their molecular composition. I will present you the general principle of the method based on a more or less accurate modeling of light propagation through scattering and absorbing media. I will give an overview of different “families" of DOT techniques, including approximated (hybrid) methods. According to the chosen instrumentation (Continuous or pulsed sources, point or wide-field illumination/detection, with/without fluorescence contrast agents...), the information content, accessible by resolution of the inverse problem, is different and will be discussed.

Thierry DAUDÉ

Non-uniqueness results for the anisotropic Calderón problem with data measured on disjoint sets.

Abstract. In this talk, we shall give some simple counterexamples to uniqueness for the anisotropic Calderón problem on Riemannian manifolds with boundary when the Dirichlet and Neumann data are measured on disjoint sets of the boundary. We provide counterexamples in the case of two and three dimensional Riemannian manifolds with boundary having the topology of circular cylinders in dimension two and toric cylinders in dimension three. The construction could be easily extended to higher dimensional Riemannian manifolds. This is joint work with Niky Kamran (McGill University) and Francois Nicoleau (Université de Nantes).

Sylvain ERVEDOZA

On the recovery of coefficients in a wave equation.

Abstract. The goal of this talk is to present recent works in which we have developed a strategy to recover a coefficient in a wave equation from one measurement of the flux. We will in particular consider the case in which the potential or the velocity is unknown. Following the setting proposed by Imanuvilov and Yamamoto in a serie of works, I will explain how one can design an algorithm to recover the coefficient of interest, in which in each step one minimizes a strictly convex quadratic functional involving Carleman weights. I will also point out the difficulties arising when adapting this strategy in the numerical setting. I will end up my talk with some numerical simulations. These results are based on joint works with Lucie Baudouin, Maya de Buhan, and Axel Osses.

Thomas GILETTI

Propagating terraces : existence and properties.

Abstract. In this talk we will discuss the dynamics of solutions of one-dimensional reaction-diffusion equations, where space-time transitions from one equilibrium to another typically occur. We will consider the general case when the profile of the propagation is not characterized by a single front, but by a layer of several fronts. This means, intuitively, that transition between equilibria may occur in several successive steps involving intermediate stationary states. In a joint work with Arnaud Ducrot and Hiroshi Matano, we introduce a notion of "propagating terrace" to describe such a situation. Our method strongly involves the so-called zero number argument, which consists in using the number of zeros of solutions (of a linear parabolic equation) as a discrete Lyapunov function.

Manuel GONZÁLEZ BURGOS

Controllability of linear parabolic systems : Hyperbolic phenomena.

Abstract. In this talk we will exhibit some new phenomena that arise in the framework of the controllability of coupled parabolic systems. When the number of (distributed or boundary) controls exerted on the system is less than the number of equations, the controllability properties of these systems develop hyperbolic phenomena as :

(1) The distributed and boundary null controllability are not equivalent.

(2) The null controllability is not equivalent to the approximate controllability.

(3) The null controllability result only holds if the final time T is large enough (minimal time of controllability).

(4) The distributed null controllability result depends on the relative position of the control open set and the support of the coupling term.

Eva SINCICH

Lipschitz stability for the electrostatic inverse boundary value problem with piecewise linear conductivities.

Abstract. We consider the electrostatic inverse boundary value problem also known as electrical impedance tomography (EIT) for the case where the conductivity is a piecewise linear function on a domain $\Omega \subset \mathbb{R}^n$ and we show that a Lipschitz stability estimate for the conductivity in terms of the local Dirichlet-to-Neumann map holds true. This is based on a joint work with G. Alessandrini, M.V. de Hoop and R. Gaburro.

Marian SLODICKA

Some inverse problems in evolutionary partial differential equations.

Abstract. We study an identification problem of a time-dependent source in a parabolic equation accompanied with various boundary conditions. We consider a bounded sufficiently smooth domain $\Omega \subset \mathbb{R}^n,\ n \geq 1$ with the boundary Γ. The inverse source problem consists of finding $\{u(x,t), h(t)\}$ obeying

$$\begin{cases} u_t(x,t) - \Delta u(x,t) &= h(t)f(x) \qquad \text{in} \ \ \Omega \times (0,T), \\ \text{Boundary conditions} & \\ u(x,0) &= u_0(x) \qquad \text{for} \ \ x \in \Omega, \end{cases} \tag{1}$$

with the final time $T > 0$. The unknown time-dependent function $h(t)$ will be determined from the following additional measurement

$$m(t) = \int_\Omega u(x,t)\ \mathrm{d}x, \qquad t \in [0,T] \tag{2}$$

or

$$m(t) = \int_\Gamma u(x,t)\ \mathrm{d}x, \qquad t \in [0,T] \tag{3}$$

The integral overdetermination (2) within the framework of inverse problems for parabolic, hyperbolic and Navier-Stokes equations has been studied in [2] and the references therein. We propose an interesting variational technique based on two steps. The first one is elimination of the unknown, the second part relies on the Rothe's method cf. [1]. We prove the well-posedness of the problem. The convergence of approximations towards the exact solution is investigated in suitable function spaces and the error estimates are derived. In the next part, we are interested in determining of the unknown couple $\{u(x,t), K(t)\}$ obeying the following semilinear parabolic problem

$$\begin{cases} \partial_t u(x,t) - \Delta u(x,t) + K(t)h(x,t) + (K * u(x))(t) &= f(x,t,u(x,t),\nabla u(x,t)), \quad \text{in } \Omega \times (0,T), \\ -\nabla u(x,t) \cdot \boldsymbol{\nu} &= g(x,t), \qquad \text{on } \Gamma \times (0,T), \\ u(x,0) &= u_0(x), \qquad \text{in } \Omega, \end{cases}$$

where $\boldsymbol{\nu}$ stands for the outer normal vector associated with Γ. The missing time-convolution kernel $K = K(t)$ will be recovered from the following integral-type measurement (2).

Final part is devoted to an inverse problem in nonlinear Maxwell's equation. The governing PDE reads as

$$\boldsymbol{E}_{tt} + \tilde{\sigma}\left(\alpha * \boldsymbol{E}\right)_t + \nabla \times \left(\mu^{-1}\nabla \times \boldsymbol{E} - \boldsymbol{f}(\boldsymbol{E})\right) = \boldsymbol{g}(\boldsymbol{E}). \tag{4}$$

We consider the following boundary condition modelling a perfect contact

$$\boldsymbol{E} \times \boldsymbol{\nu} = \boldsymbol{0} \qquad \text{on } \Gamma \tag{5}$$

and the initial data

$$\boldsymbol{E}(\boldsymbol{x}, 0) = \boldsymbol{E}_0(\boldsymbol{x}), \qquad \boldsymbol{E}_t(\boldsymbol{x}, 0) = \boldsymbol{V}_0(\boldsymbol{x}). \tag{6}$$

The inverse problem is to find a couple $\{\boldsymbol{E}(\boldsymbol{x}, t), \alpha(t)\}$ from

$$\int_\Gamma \boldsymbol{E} \cdot \boldsymbol{\nu} \, \mathrm{d}\gamma = m(t), \qquad \text{(normal component measurement)}. \tag{7}$$

Références

[1] Kačur, J. *Method of Rothe in evolution equations*, Volume 80 of *Teubner Texte zur Mathematik.* Leipzig : Teubner,1985

[2] A.I. Prilepko, D.G. Orlovsky, and I.A. Vasin. *Methods for solving inverse problems in mathematical physics.* Pure and Applied Mathematics, Marcel Dekker. 231. New York, NY : Marcel Dekker, 2000.

Masahiro YAMAMOTO

Coefficient inverse problems for integro-partial differential equations by Carleman estimates.

Abstract. We consider several model equations for the viscoelasticity such as the Kelvin-Voigt model. These equations are parabolic or hyperbolic type of equations with integral terms. We show several Carleman estimates and apply them to establish the conditional stability in determining spatially varying Lame coefficients.

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