# Problèmes inverses et domaines associés

3 et 4 décembre 2015
FRUMAM, Marseille

## Présentation

Comme en 2013 et 2014, nous organisons dans le cadre du groupe de travail GOMS, et avec le soutien du groupe 3 du CPT, de l'équipe Analyse Appliquée de l'I2M, et du groupe de travail "Calcul des variations et EDP" de la FRUMAM, deux journées thématiques sur les problèmes inverses, le contrôle et l'analyse des EDP, les 3 et 4 décembre 2015 à la FRUMAM.

## Conférenciers

Giovanni ALBERTI (Department of Mathematics, ETH Zürich)
Cédric BELLIS (Laboratoire de Mécanique et d'Acoustique, CNRS, Marseille)
Camille LAURENT (Laboratoire Jacques-Louis Lions, CNRS, Paris)
Alexandre MUNNIER (Institut Élie Cartan de Lorraine, Université de Lorraine, Nancy)
Lauri OKSANEN (University College London Centre for Inverse Problems)
Erica SCHWINDT (Laboratoire Jacques-Louis Lions, UPMC, Paris)
Faouzi TRIKI (Laboratoire Jean Kuntzmann, Université Joseph Fourier, Grenoble)
Masahiro YAMAMOTO (Department of Mathematical Sciences, Tokyo University)

## Programme

### Jeudi 3 décembre

| | |
|---|---|
| **12h00 − 14h00** | **Accueil des participants.** |
| **14h00 − 15h00** | **Cédric BELLIS** |
| | Acoustic inverse scattering using topological derivative of far-field measurements-based $L^2$ cost functionals. |
| **15h00 − 16h00** | **Giovanni ALBERTI** |
| | Non-zero constraints in PDE and applications to hybrid inverse problems. |
| **16h00 − 16h20** | **Pause café** |
| **16h20 − 17h20** | **Alexandre MUNNIER** |
| | Reconstructing a two dimensional cavity from boundary measurements. |
| **20h00** | **Diner** |

## Vendredi 5 décembre

| | |
|---|---|
| **9h00 − 10h00** | **Camille LAURENT** |
| | Quantitative unique continuation for operators with partially analytic coefficients. Application to approximate control for waves. |
| **10h − 10h20** | **Pause café** |
| **10h20 − 11h20** | **Lauri OKSANEN** |
| | Inverse problems for the connection Laplacian. |
| **11h20 − 12h20** | **Erica SCHWINDT** |
| | On the uniqueness and stability of an inverse problem in photo-acoustic tomography. |
| **12h20 − 14h00** | **Déjeuner** |
| **14h00 − 15h00** | **Masahiro YAMAMOTO** |
| | TBA. |
| **15h00 − 16h00** | **Faouzi TRIKI** |
| | Photoacoustics imaging in stratified media. |

# Résumés des exposés

## Giovanni ALBERTI

**Title** : Non-zero constraints in PDE and applications to hybrid inverse problems

**Abstract** : In this talk I will describe a multiple frequency approach to the boundary control of Helmholtz and Maxwell equations. We give boundary conditions and a finite number of frequencies such that the corresponding solutions satisfy certain non-zero constraints inside the domain. The suitable boundary conditions and frequencies are explicitly constructed and do not depend on the coefficients, in contrast to the illuminations given as traces of complex geometric optics solutions.
This theory finds applications in several hybrid imaging modalities : these constraints are needed to prove stability and to apply explicit reconstruction formulae. Similarly, multiple frequencies guarantee uniqueness and stability for the linearised inverse problem in acousto-electromagnetic tomography, thereby obtaining the convergence of a Landweber iteration scheme.

## Cédric BELLIS

**Title** : Acoustic inverse scattering using topological derivative of far-field measurements-based $L^2$ cost functionals.

**Abstract** : Originally formulated in the context of topology optimization, the concept of topological derivative has also proved effective as a qualitative inversion tool for wave-based identification of finite-sized objects. This approach remains however largely based on a heuristic interpretation of the topological derivative, whereas most other qualitative approaches to inverse scattering are backed by a mathematical justification. As an effort towards bridging this gap, this study focuses on a topological derivative approach applied to the $L^2$-norm of the misfit between far-field measurements. Either an inhomogeneous medium or a finite number of point-like scatterers are considered, using either the Born approximation or a full scattering model. Topological derivative-based imaging functionals are analyzed using a suitable factorization of the far-field operator, for each of the considered cases, in order to characterize their behavior and assess their ability to reconstruct the unknown scatterer(s). Results include the justification of the usual sign heuristic underpinning the method for (i) the Born approximation and (ii) full-scattering models limited to moderately strong scatterers. Semi-analytical and numerical examples are presented. Within the chosen framework, the topological derivative approach is finally discussed and compared to other well-known qualitative methods.

**Camille LAURENT**

**Title** : Quantitative unique continuation for operators with partially analytic coefficients. Application to approximate control for waves.

**Abstract** : Unique continuation is very often proved by Carleman estimates or Holmgren theorem. The first one requires the strong geometric assumption of pseudoconvexity of the hypersurface. The second one only requires that the hypersurface is non characteristic, but the coefficients need to be analytic.
Motivated by the example of the wave equation, several authors (Tataru, Robbiano-Zuily, Hörmander) finally proved in great generality that there could be unique continuation in some intermediate situation where the coefficients are analytic in part of the variables. In particular, for the wave equation, it allowed to prove the unique continuation across any non characteristic hypersurface for non analytic metric.
In this talk, after presenting these works, I will describe some recent work where we quantify this unique continuation. This leads to the optimal (in general) logarithmic stability estimates. We will also give some applications to controllability.

**Alexandre MUNNIER**

**Title** : Reconstructing a two dimensional cavity from boundary measurements.

**Abstract** : In this work, we address a classical case of the Calderón (or conductivity) inverse problem in dimension two. We aim to recover the location and the shape of a single cavity $\omega$ (with boundary $\gamma$) contained in a domain $\Omega$ (with boundary $\Gamma$) from the knowledge of the Dirichlet-to-Neumann (DtN) map $\Lambda_\gamma : f \longmapsto \partial_n u^f|_\Gamma$, where $u^f$ is harmonic in $\Omega \setminus \overline{\omega}$, $u^f|_\Gamma = f$ and $u^f|_\gamma = c^f$, $c^f$ being the constant such that $\int_\gamma \partial_n u^f \, \mathrm{d}s = 0$. We obtain an explicit formula for the complex coefficients $a_m$ arising in the expression of the Riemann map $z \longmapsto a_1 z + a_0 + \sum_{m \leqslant -1} a_m z^m$ that conformally maps the exterior of the unit disk onto the exterior of $\omega$. This formula is derived by using two ingredients : a new factorization result of the DtN map and the so-called generalized Pólia-Szegö tensors (GPST) of the cavity. As a byproduct of our analysis, we also prove the analytic dependence of the coefficients $a_m$ with respect to the DtN. Numerical results will be provided to illustrate the efficiency and simplicity of the method.

**Lauri OKSANEN**

**Title** : Inverse problems for the connection Laplacian.

**Abstract** :We reconstruct a Riemannian manifold and a Hermitian vector bundle with compatible connection from the hyperbolic Dirichlet-to-Neumann operator associated with the wave equation of the connection Laplacian. The boundary data is local and the reconstruction is up to the natural gauge transformations of the problem. As a corollary we derive an elliptic analogue of the main result which solves a Calderon problem for connections on a cylinder. The talk is based on joint work Yaroslav Kurylev (University College London) and Gabriel P. Paternain (University of Cambridge).

**Erica SCHWINDT**

**Title** : On the uniqueness and stability of an inverse problem in photo-acoustic tomography.

**Abstract** : In this talk, I will present an inverse problem in photo-acoustic tomography. The aim is to recover and characterize the absorption coefficient of a soft body. The inverse problem is formulated as a problem of optimal control in which the control variable is the coefficient to retrieve.
The result of existence of at least one optimal control was proved in " An optimal control problem in photoacoustic tomography " by M. Bergounioux et al.
In this presentation, I will deal with the problem of the uniqueness of the optimal solution (absorption coefficient) and also I will present a study on the sensitivity of this solution with respect to variations of the source of illumination and with respect to observation.

**Faouzi TRIKI**

**Title** : Photoacoustics imaging in stratified media.

**Abstract** : TBA

**Masahiro YAMAMOTO**

**Title** : TBA.

**Résumé** : TBA

# Adresse électronique des conférenciers et des organisateurs

| | |
|---|---|
| Giovanni ALBERTI | ETH Zürich |
| | giovanni.alberti@sam.math.ethz.ch |
| Cédric BELLIS | CNRS, Marseille |
| | bellis@lma.cnrs-mrs.fr |
| Michel CRISTOFOL | Aix-Marseille Université |
| | michel.cristofol@univ-amu.fr |
| Camille LAURENT | CNRS, Paris |
| | laurent@ann.jussieu.fr |
| Alexandre MUNNIER | Université de Lorraine |
| | alexandre.munnier@univ-lorraine.fr |
| Lauri OKSANEN | Université College London |
| | l.oksanen@ucl.ac.uk |
| Erica SCHWINDT | Université Pierre et Marie Curie |
| | schwindt@math.cnrs.fr |
| Faouzi TRIKI | Université Joseph Fourier |
| | faouzi.triki@imag.fr |
| Eric SOCCORSI | Aix-Marseille Université |
| | eric.soccorsi@univ-amu.fr |
| Masahiro YAMAMOTO | Tokyo University |
| | myama@next.odn.ne.jp |