Séminaire *Guides d'ondes, milieux stratifiés et problèmes inverses*

Frumam, Marseille

Mercredi 23 mars 2016

• 14h00-15h00 : GIUSEPPE CARDONE (University del Sannio)

Uniform resolvent convergence for a strip with fast oscillating boundary

In a planar infinite strip with a fast oscillating boundary we consider an elliptic operator assuming that both the period and the amplitude of the oscillations are small. On the oscillating boundary we impose Dirichlet, Neumann or Robin boundary condition. In all cases we describe the homogenized operator, establish the uniform resolvent convergence of the perturbed resolvent to the homogenized one, and prove the estimates for the rate of convergence. These results are obtained as the order of the amplitude of the oscillations is less, equal or greater than that of the period. It is shown that under the homogenization the type of the boundary condition can change.

• 15h30-16h30 : ANDRII KHRABUSTOVSKYI (Karlsruhe Institute of Technology)

Periodic differential operators with predefined spectral gaps

It is well-known that the spectrum of self-adjoint periodic differential operators has a band structure, i.e. it is a locally finite union of compact intervals called *bands*. In general the bands may overlap. The bounded open interval $(a, b) \subset \mathbb{R}$ is called a *gap* in the spectrum of the operator $\mathcal{H}$ if $(a, b) \cap \mathcal{H} = \emptyset$ and $a, b \in \sigma(\mathcal{H})$.

The presence of gaps in the spectrum is not guaranteed: for example, the spectrum of the Laplacian in $L^2(\mathbb{R}^n)$ has no gaps, namely $\sigma(-\Delta_{\mathbb{R}^n}) = [0, \infty)$. Therefore the natural problem is a construction of periodic operators with non-void spectral gaps. The importance of this problem is caused by various applications, for example in physics of photonic crystals. We refer to the overview [1], where a lot of examples are discussed in detail.

Another important question arising here is how to control the location of the gaps via a suitable choice of the coefficients of the operators or/and via a suitable choice of the geometry of the medium. In the talk we give an overview of the results obtained in [2, 3, 4, 5], where this problem is studied for various classes of periodic differential operators.

In a nutshell, our goal is to construct an operator (from some given class of periodic operators) such that its spectral gaps are close (in some natural sense) to predefined intervals.

REFERENCES

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[2] A. Khrabustovskyi, Periodic Riemannian manifold with preassigned gaps in spectrum of Laplace-Beltrami operator, Journal of Differential Equations, 252(3) (2012), 2339–2369.

[3] A. Khrabustovskyi, Periodic elliptic operators with asymptotically preassigned spectrum, Asymptotic Analysis, 82(1-2) (2013), 1-37.

[4] A. Khrabustovskyi, Opening up and control of spectral gaps of the Laplacian in periodic domains, Journal of Mathematical Physics, 55(12) (2014), 121502.

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